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THE NATIONAL UNIVERSITY OF IRELAND, CORK

COLÁISTE NA hOLLSCOILE, CORCAIGH  
UNIVERSITY COLLEGE, CORK

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SUMMER EXAMINATIONS, 2005

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B.E. DEGREE (ELECTRICAL)

MECHATRONICS AND INDUSTRIAL AUTOMATION  
EE4009

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Time Allowed: 3 hours

Answer five questions.

All questions carry equal marks.

The use of a Casio fx570w or fx570ms calculator is permitted.

1. A SCARA robot is to execute a three-point trajectory in Cartesian space using piecewise interpolation with a parabolic blend. The total time of the traverse is to be 18 s, the blend time,  $\Delta T$ , is 1 s on each side of the knot point, the tool velocity is to be the same for the two linear sections and the points are given by tool-configuration vectors:

$$\begin{aligned}w^1 &= [4, 3, 0, 0, 0, -1, 0.8]^T \\w^2 &= [0, 0, 0, 0, 0, -1, 0.5]^T \\w^3 &= [4, 0, 0, 0, 0, -1, 0.8]^T\end{aligned}$$

- (a) Find the  $y$  component of the trajectory  $w(t)$ , over the time interval  $[0, 20]$ . [11 marks]  
(b) What is the deviation of the tool roll angle from its desired value when the tool is in the vicinity of the knot point  $w^2$ ? [3 marks]  
(c) Given that the elbow joint angle for the SCARA robot is defined by

$$\Theta_2 = \pm \text{Arccos} \left( \frac{p_x^2 + p_y^2 - a_1^2 - a_2^2}{2a_1a_2} \right)$$

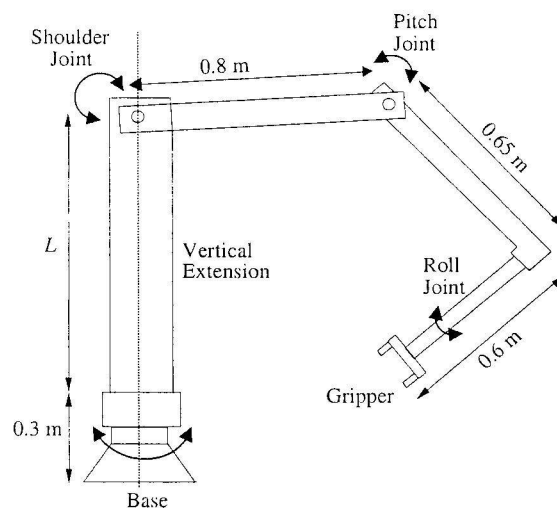
(with  $a_1 = a_2 = 5$  in this case), describe how the angular velocity of the elbow joint varies over the last 6 s of the trajectory, i.e. as it approaches  $w^3$ . [6 marks]

2. For each the following types of work cells:

- (a) Robot-centred cell, [4 marks]
- (b) In-line cell, [12 marks]
- (c) Mobile robot cell, [4 marks]

describe the operation of the cell, highlighting any major advantages and disadvantages. For in-line cells, treat cells with intermittent, continuous and non-synchronous transfer mechanisms separately. Use diagrams to illustrate your answer, as appropriate, and include a typical application of each of the three major cell types.

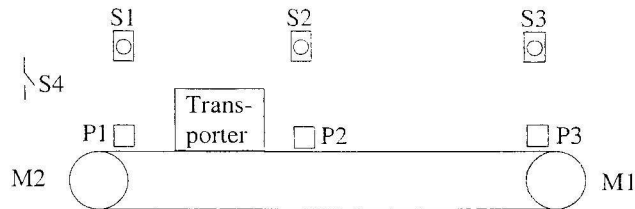
3. A five axis robot has a revolute base joint followed by a vertical prismatic axis and shoulder, pitch and roll revolute joints, as shown below. This robot has no offsets in a direction perpendicular to the plane of the paper. The shoulder and pitch joints have a horizontal axis. A gripper acts as the end effector, as shown. The prismatic joint is such that the length  $L$  can vary from 0.4 m to 0.85 m.



If the dimensions of the system are as on the diagram, use the Denavit-Hartenberg algorithm and matrix (available as Appendix I at the end of this paper) to

- (a) Assign suitable frames to the robot, using a link-coordinate diagram. It is not necessary to show the  $y$  axes of frames, except where required in part (d) below. [11 marks]
- (b) Tabulate the kinematic parameters associated with the robot. [5 marks]
- (c) Write down an expression for  $T_{base}^{tool}$  for this robot, based on your frame assignments. Your answer can take the form of a product of a number of matrices, i.e. you are *not* required to perform the multiplication. [2 marks]
- (d) Perform a sanity check on  $T_3^4$ . [2 marks]

4. An automated vehicle is used to cyclically move between three points, stopping at designated points when required.  
Use a ladder diagram to show how a PLC could be used to control the process.



*Equipment description:*

The transporter is operated using a bidirectional conveyor belt. A motor M1 is activated to move the conveyor from left to right, while a motor M2 moves it in the opposite direction. Proximity sensors, P1 to P3, (closed implies object present) are used to detect when the transporter is at those positions where it may be required to stop. Non-latching switches (closed when pushed), S1 to S3, are located near P1 to P3 for signalling purposes

A latched on/off switch S4 is also provided.

*Operational Specifications:*

- (a) The system is enabled when S4 is closed.
  - (b) The transporter moves in the sequence P1 to P2 to P3 to P1. It should move continuously except at the points P1 to P3, and it will only stop at those points for which the associated non-latching switch has been pushed since the transporter was last at that point. The stops at P1, P2 and P3 should be for 20 s, 30 s, and 40 s, respectively.
  - (c) The initial direction of motion is not important. You need not include special provisions for start-up. [20 marks]
5. Describe the operation of each of the following in determining both the position and velocity of a rotating machine shaft.
- (a) an incremental angular encoder, [10 marks]
  - (b) a resolver (with resolver to digital converter). [10 marks]

Include diagrams to illustrate your answer. Derivations of resolver-to-digital transfer functions are *not* required.

6. The forward kinematic equations of a SCARA robot are as follows:

$$\begin{aligned}
 T_{\text{base}}^{\text{tool}} &= \begin{bmatrix} C_1 & S_1 & 0 & a_1 C_1 \\ S_1 & -C_1 & 0 & a_1 S_1 \\ 0 & 0 & -1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_2 & -S_2 & 0 & a_2 C_2 \\ S_2 & C_2 & 0 & a_2 S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
 &\quad \times \begin{bmatrix} C_4 & -S_4 & 0 & 0 \\ S_4 & C_4 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} C_{1-2-4} & S_{1-2-4} & 0 & a_1 C_1 + a_2 C_{1-2} \\ S_{1-2-4} & -C_{1-2-4} & 0 & a_1 S_1 + a_2 S_{1-2} \\ 0 & 0 & -1 & d_1 - q_3 - d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}
 \end{aligned}$$

where  $q_3$  is the variable joint offset of the prismatic joint.

(a) Derive the inverse kinematic equations of the SCARA robot.

(You are expected to derive the equation relating to the elbow joint angle from first principles; i.e. any identity used must be derived using trigonometrical *or other methods*.)

[13 marks]

(b) Derive the elements of the second *column* of the manipulator Jacobian of this robot, assuming the information on forward kinematics given above. (There is no need to give a detailed description of the general algorithm for the computation of Jacobian matrices).

[7 marks]

7. A vision system is to be used in an automation application. Both the focal length,  $f$ , of the camera and its position relative to a base origin are unknown. However, the orientation of the camera frame is known. Specifically, it can be assumed that

$$T_{\text{base}}^{\text{camera}} = \begin{bmatrix} 1 & 0 & 0 & x_0 \\ 0 & -1 & 0 & y_0 \\ 0 & 0 & -1 & z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where  $x_0, y_0, z_0$  are not known, and that

$$T_{\text{camera}}^{\text{base}} = \begin{bmatrix} 1 & 0 & 0 & -x_0 \\ 0 & -1 & 0 & +y_0 \\ 0 & 0 & -1 & +z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Points  $M$  and  $N$  are used for camera calibration purposes and are specified in the base frame. Points  $M^i$  and  $N^i$  are the images of  $M$  and  $N$ . Note that the two test points are at different heights relative to the base. Specifically, note that  $N$  has an  $z$  co-ordinate of 20 cm relative to the base.

The test points (defined in the base frame, all distances being given in cm) are  $M = [30, 0, 0]^T$  and  $N = [50, 0, 20]^T$ . These have images  $M^i = [-0.5263, -1.5789, 0]^T$  and  $N^i = [-2.0, -2.0, 0]^T$  in the camera frame. Calculate the four unknowns:  $f, x_0, y_0$  and  $z_0$ .

(*Hint:* Derive four equations relating the  $x$  and  $y$  values of each of the images  $M^i$  and  $N^i$  to the focal length of the camera,  $f$ , the coordinates of the test points and the camera position values, and solve for the unknowns.)

[20 marks]

## Appendix I

### Denavit-Hartenberg Algorithm and Matrix

1. Number the joints from 1 to  $n$  starting with the base and ending with the tool yaw, pitch and roll, in that order.
2. Assign a right-handed orthonormal coordinate frame  $L_0$  to the robot base, making sure that  $z^0$  aligns with the axis of joint 1. Set  $k = 1$ .
3. Align  $z^k$  with the axis of joint  $k + 1$ .
4. Locate the origin of  $L_k$  at the intersection of the  $z^k$  and  $z^{k-1}$  axes. If they do not intersect, use the intersection of  $z^k$  with a common normal between  $z^k$  and  $z^{k-1}$ .
5. Select  $x^k$  to be orthogonal to both  $z^k$  and  $z^{k-1}$ . If  $z^k$  and  $z^{k-1}$  are parallel, point  $x^k$  away from  $z^{k-1}$ .
6. Select  $y^k$  to form a right-handed orthonormal coordinate frame  $L_k$ .
7. Set  $k = k + 1$ . If  $k < n$ , go to step 3; else, continue.
8. Set the origin of  $L_n$  at the tool tip. Align  $z^n$  with the approach vector,  $y^n$  with the sliding vector, and  $x^n$  with the normal vector of the tool. Set  $k = 1$ .
9. Locate point  $b^k$  at the intersection of the  $x^k$  and  $z^{k-1}$  axes. If they do not intersect, use the intersection of  $x^k$  with a common normal between  $x^k$  and  $z^{k-1}$ .
0. Compute  $\Theta_k$  as the angle of rotation from  $x^{k-1}$  to  $x^k$  measured about  $z^{k-1}$ .
1. Compute  $d_k$  as the distance from the origin of frame  $L_{k-1}$  to point  $b_k$  measured along  $z^{k-1}$ .
2. Compute  $a_k$  as the distance from point  $b^k$  to the origin of frame  $L_k$  measured along  $x^k$ .
3. Compute  $\alpha_k$  as the angle of rotation from  $z^{k-1}$  to  $z^k$  measured about  $x^k$ .
4. Set  $k = k + 1$ . If  $k \leq n$ , go to step 9; else, stop.

ENAVIT-HARTENBERG MATRIX:

$$T_{i-1}^i = \begin{bmatrix} C\Theta_i & -S\Theta_i C\alpha_i & S\Theta_i S\alpha_i & a_i C\Theta_i \\ S\Theta_i & C\Theta_i C\alpha_i & -C\Theta_i S\alpha_i & a_i S\Theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$