

**OLLSCOIL NA hÉIREANN, CORCAIGH**  
THE NATIONAL UNIVERSITY OF IRELAND, CORK

COLÁISTE NA hOLLSCOILE, CORCAIGH  
UNIVERSITY COLLEGE, CORK

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**SUMMER EXAMINATIONS, 2004**

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**B.E. DEGREE (ELECTRICAL)**  
**HIGHER DIPLOMA IN PHYSICS**

OPTICAL ELECTRONICS  
EE4007

Professor G.W. Irwin  
Professor P.J. Murphy  
Dr. S.L. Prunty  
Dr. A.P. Morrison

Time Allowed: *3 hours*

FIVE QUESTIONS TO BE ANSWERED, AT LEAST TWO FROM EACH SECTION.  
USE SEPARATE ANSWER BOOKS FOR EACH SECTION

The use of a Casio fx570w or fx570ms calculator is permitted.  
The use of Log Tables and Graph paper are permitted.

*Physical Constants:*

Free electron mass,  $m_0 = 9 \times 10^{-31}$  kg  
Planck's constant,  $h = 6.626 \times 10^{-34}$  J s  
Electronic charge,  $q = 1.602 \times 10^{-19}$  C  
Boltzmann's constant,  $k_B = 1.38 \times 10^{-23}$  J K<sup>-1</sup>  
Room temperature = 300 K  
Speed of light in free space,  $c = 3 \times 10^8$  m s<sup>-1</sup>

## SECTION A

1. (a) Write coupled rate equations for the population inversion and the photon number for an idealized 4-level laser system (assume that the relaxation from the lower laser level is so fast that the approximation  $N_1 \approx 0$  applies). Justify the inclusion of the various terms appearing in your equations.

[6 marks]

- (b) Under steady state conditions, derive an expression for the threshold population inversion, and show also that the population inversion (well above the threshold for laser oscillation) remains clamped at the threshold value.

[4 marks]

- (c) A Nd<sup>3+</sup>:YAG laser has the following parameters:  $\lambda=1.06 \mu\text{m}$ ,  $t_{\text{spont}}=0.55 \text{ ms}$ ,  $\Delta\nu=120 \text{ GHz}$ , and refractive index  $n=1.82$ . Assuming a cavity lifetime ( $\tau_c$ ) of 20 ns, find the threshold population inversion density.

[6 marks]

- (d) Determine the power in  $\text{W.cm}^{-3}$  given off by spontaneous emission just below threshold in (c) above.

[4 marks]

2. (a) By considering the process of amplification in an inverted medium, show that an electromagnetic wave traversing a slab of the medium of thickness  $\delta z$  will grow in intensity according to:

$$\frac{dI_\nu}{dz} = \gamma(\nu)I_\nu,$$

where  $\nu$  denotes the frequency. Show that  $\gamma(\nu)$  is proportional to the population inversion and determine the proportionality factor.

[8 marks]

- (b) A CO<sub>2</sub> laser transition at  $10.6 \mu\text{m}$  has the following characteristics:  $A_{21}=0.34 \text{ s}^{-1}$  and  $\Delta\nu=1 \text{ GHz}$ .

- i. What is the stimulated emission cross-section at line centre? [4 marks]
- ii. What must be the population inversion to obtain a gain coefficient of  $0.05 \text{ cm}^{-1}$ ? [4 marks]
- iii. What is the saturation intensity ( $I_s$ ) if the upper state lifetime ( $\tau_2$ ) is  $10 \mu\text{s}$ ? [4 marks]

3. (a) Write the ray transfer matrix for a thin convex lens of focal length  $f$ . [2 marks]

- (b) The complex beam parameter of a Gaussian beam is given by:

$$\frac{1}{q(z)} = \frac{1}{R(z)} + j \cdot \frac{\lambda}{\pi \omega^2(z)},$$

Using the ABCD law, derive expressions for  $R(z)$  and  $\omega(z)$  at an arbitrary plane in an optical cavity by forcing  $q(z)$  to transform into itself after a round trip.

[6 marks]

- (c) Use the results in (b) above to write an equation for the stability of the cavity, and show that the equality  $A=D$  for the diagonal elements of the ray transfer matrix corresponds to the position of the beam waist.

[4 marks]

- (d) An optical cavity is formed by placing a thin convex lens of focal length  $f$  between two plane mirrors. The distance of the lens from one of the plane mirrors is  $d_1$  while the distance from the other is  $d_2$ . Determine the ray matrix for the unit cell that starts at one of the plane mirrors. Using this matrix, determine the range of location of the lens for stability of the cavity.

[8 marks]

4. (a) Briefly describe the technique of Q-switching as applied to laser systems and list some applications of the technique.

[5 marks]

- (b) A Ruby rod for a Q-switched laser ( $\lambda=0.6943 \mu\text{m}$ ) is 10 cm long and 3 mm radius, and is placed in an optical cavity 30 cm long. The  $\text{Cr}^{3+}$  doping density is  $1.6 \times 10^{19} \text{ cm}^{-3}$  and the stimulated emission cross-section is  $1.3 \times 10^{-20} \text{ cm}^2$ . The pumping agent creates an initial population of  $10^{19} \text{ atoms cm}^{-3}$  in the upper laser state. Assume that the pumping occurs by virtue of absorption at  $\lambda=0.45 \mu\text{m}$ , and that 100% of the atoms pumped to higher states relax to state 2. The spontaneous lifetime of state 2 is 3 ms. The index of refraction of the ruby rod is 1.78 and the mirrors have reflectivities of 0.95 and 0.7 with negligible absorption losses. The 0.95 reflectivity mirror has zero transmission.

- i. How much spontaneous power does the ruby crystal radiate before the Q-switch is opened?  
[3 marks]
- ii. How much pump power is required to maintain the population in state 2 at  $10^{19} \text{ atoms cm}^{-3}$ ?  
[3 marks]
- iii. What is the peak output power of the Q-switched pulse?  
[3 marks]
- iv. How much energy is contained in the output pulse? (Assume that the energy extraction efficiency is 90%)  
[3 marks]
- v. Estimate the duration of the output pulse.  
[3 marks]

## SECTION B

5. (a) Calculate the link margin for the system specified in the diagram below. [8 marks]

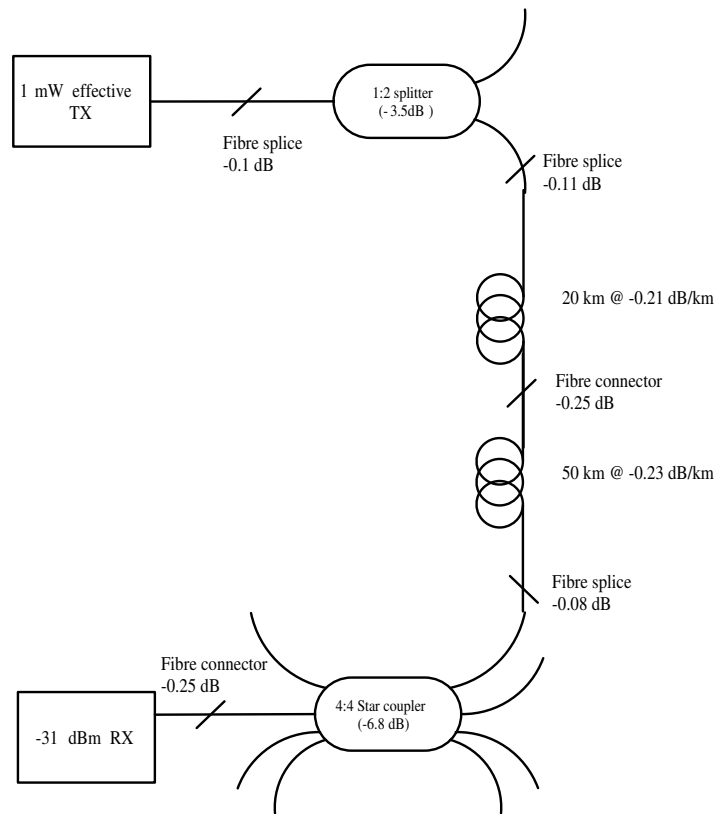


Figure 1:

- (b) Explain the following terms:

- i. Modal distortion
- ii. material dispersion
- iii. soliton
- iv. Fibre attenuation
- v. Link margin

[5 marks]

- (c) If the material dispersion for silica is 110 ps/nm.km at 820 nm and 15 ps/nm.km at 1.5  $\mu\text{m}$  determine whether it is more advantageous to use an 820 nm LED having a spectral linewidth of 10 nm or a 1550 nm LED having a 60 nm spectral width. Justify your answer numerically. What will be the total pulse spreading for each LED if the link length is 25 km? [7 marks]

6. (a) It is required to design an integrated optic directional coupler with two thirds of the power going to one output port and one third to the other. The coupling length is given as 3 cm. What should be the length of the coupling region? [7 marks]

[7 marks]

- (b) A fibre has an  $NA = 0.2588$ . A light source is coupled to this that will emit 75 % of its light into a  $60^\circ$  full-cone angle, 50 % into a  $30^\circ$  cone and 25 % into a  $15^\circ$  cone. What is the coupling efficiency when this source and fibre are connected?  
[6 marks]
- (c) A fibre has a core refractive index of 1.5 and a cladding refractive index of 1.49, and a core diameter of  $50 \mu\text{m}$ . Consider the guided ray travelling at the steepest angle with respect to the fibre axis. How many reflections are there per meter for this ray?  
[7 marks]
7. (a) List five differences between Light-emitting diodes (LEDs) and Laser diodes.  
[4 marks]
- (b) Show using diagrams the physical effect compressive strain and tensile strain has on a mismatched epi-layer. Is the epi-layer lattice constant greater or smaller than that of the substrate for tensile strain to occur?  
[4 marks]
- (c) List suitable semiconductor materials for light emission at wavelengths around 650 nm, 780 nm, and 1300 nm. (one of each).  
[3 marks]
- (d) ) A GaAs/AlGaAs LED has an activation barrier of energy  $E_d = 0.6 \text{ eV}$  to long-term degradation. If the prefactor in the Arrhenius equation describing the degradation rate is  $C = 250 \text{ hour}^{-1}$ , find the time after which the output radiant power will fall to half its initial value, assuming room temperature ( $25^\circ \text{ C}$ ) operation.  
[9 marks]
8. (a) Explain the term Noise Equivalent Power (NEP) with reference to semiconductor photodetectors.  
[5 marks]
- (b) Describe the process of impact ionization as applied to avalanche photodiodes. How does this provide internal gain? What is the advantage of internal gain in the photodiode?  
[5 marks]
- (c) A photodiode having a circular active area of 1 cm radius has a responsivity of  $0.55 \text{ A/W}$  to light at 633 nm. The noise equivalent power for the photodiode is  $2.5 \times 10^{-12} \text{ W Hz}^{-1/2}$ . This photodiode is to be used in a LIDAR (Light detection and ranging) system in conjunction with a He-Ne laser having a non-divergent beam with 5 mW average output power. Assume no attenuation of the laser light, a bandwidth of 1 Hz and assume the target is a perfect diffuse reflector (i.e. reflected light is scattered uniformly in all directions) - what is the maximum target distance that can be measured in this system? What is the photocurrent produced?  
[10 marks]