OLLSCOIL NA hÉIREANN, CORCAIGH<br>THE NATIONAL UNIVERSITY OF IRELAND, CORK<br>COLÁISTE NA hOLLSCOILE, CORCAIGH<br>UNIVERSITY COLLEGE, CORK<br>SUMMER EXAMINATIONS, 2006<br>B.E. DEGREE (ELECTRICAL)<br>B.E. DEGREE (MICROELECTRONIC)<br>M. ENG. SC. DEGREE (MICROELECTRONIC)<br>TELECOMMUNICATIONS<br>EE4004<br>Professor Dr. U. Schwalke<br>Professor P. J. Murphy<br>Dr. K. G. McCarthy<br>Mr. C. Murphy<br>Time allowed: 3 hours<br>Answer five questions.

The use of log tables and a Casio fx570w or fx570ms calculator is permitted.

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Q.1. (a) Describe how non-linear quantization is used to minimize quantization noise in PCM systems and, in that context, discuss the $A$-law and $\mu$-law implementations.
[7 marks]
(b) A microwave link using 256 QAM modulation is used to carry three signals of 5 $\mathrm{MHz}, 10 \mathrm{MHz}$ and 15 MHz bandwidth respectively, which have been encoded using PCM. If the carrier modulation rate is $6 \times 10^{7}$ phase changes per second, what is the maximum possible signal to quantisation noise ratio in dB ?
(c) Discuss the use of line codes in PCM telephony systems.
Q.2. (a) Using the GSM system as an example, discuss the concept of logical channels in digital mobile phone networks.
[10 marks]
(b) Describe the concept and operation of statistical multiplexing in data communications links. Include a definition of multiplexing gain in your discussion and indicate the types of data sources that are likely to give rise to both low and high multiplexing gain.
[10 marks]
Q.3. (a) Explain the concept of transparent routing when interconnecting LANs and illustrate how this may give rise to loops using an appropriate diagram.
[5 marks]
(b) The network below consists of 6 LANs (a to e) interconnected by means of 6 bridges (B1 to B6). The spanning tree algorithm is used to determine the shortest paths in the network. Describe and illustrate each step of the algorithm as well as the resulting shortest paths.

[15 marks]
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Q.4. A binary symmetric channel ( BSC ) with probability of error $p_{1}\left(p_{1} \neq 1 / 2\right)$ is connected in series with a second BSC (i.e. the outputs of the first BSC become the inputs of the second) having probability of error $p_{2}\left(p_{2} \neq 1 / 2\right)$. Show that if this arrangement is represented by a single equivalent system having inputs $x_{0}, x_{1}$ and corresponding outputs $y_{0}, y_{1}$ then: -
(a) The equivalent system is also a BSC with probability of error, denoted $p_{E}$, given by:

$$
p_{E}=p_{1}+p_{2}-2 p_{1} p_{2} .
$$

(b) The entropy of the output, denoted $H[Y]$, is given by: -

$$
H[Y]=-\left(a \log _{2}[a]+(1-a) \log _{2}[1-a]\right)
$$

where $a=p_{x_{0}}+p_{E}\left(1-2 p_{x_{0}}\right)$ and $p_{x_{0}}$ denotes the probability of sending symbol $x_{0}$.
(c) The channel capacity of the equivalent system is given by: -

$$
\begin{gathered}
C_{s}=1+\left(1+2 p_{1} p_{2}-p_{1}-p_{2}\right) \log _{2}\left[1+2 p_{1} p_{2}-p_{1}-p_{2}\right]+ \\
\left(p_{1}+p_{2}-2 p_{1} p_{2}\right) \log _{2}\left[p_{1}+p_{2}-2 p_{1} p_{2}\right] .
\end{gathered}
$$

[8 marks]
(d) If $p_{1}$ and $p_{2}$ are sufficiently small such that $p_{E} \approx p_{1}+p_{2}$, deduce the corresponding approximation for the channel capacity $C_{s}$.
Q.5. A binary modulation scheme is described by:-

$$
s_{i}(t)= \begin{cases}s_{1}(t)=A_{1} \cos \left(\omega_{c} t\right) & 0 \leq t \leq T \\ s_{2}(t)=A_{2} \cos \left(\omega_{c} t\right) & 0 \leq t \leq T\end{cases}
$$

where $T$ is an integer times $1 / f_{c}$. For this modulation scheme, given that (under the usual assumptions) $P_{e}=Q\left[\sqrt{\frac{E_{d}}{2 \eta}}\right]$, show that: -
(a) $P_{e}=Q\left[\sqrt{\frac{\left(A_{1}-A_{2}\right)^{2} T}{4 \eta}}\right]$.
[8 marks]
(b) If the average signal energy per bit (denoted $E_{b}$ ) is a fixed constant and $T$ and $\eta$ are also constant, prove that $P_{e}$ in part (a) above is minimized if $A_{2}=-A_{1}$. Hint: the minimum value of $Q[\sqrt{x}]$ occurs when $x$ takes on its maximum possible value.
[12 marks]
Q.6. Using the primitive polynomial $p(x)=x^{4}+x+1$, generate the field $G F\left(2^{4}\right)$.
(b) Given that the generator polynomial for the $(15,7)$ double error correcting primitive BCH code based upon this field, denoted $g(x)$, is given by: -

$$
g(x)=x^{8}+x^{7}+x^{6}+x^{4}+1
$$

and a particular error free non-systematic code word, denoted $c(x)$, is given by:

$$
c(x)=x^{11}+x^{8}+x^{7}+x^{6}+x^{3}+x^{2},
$$

deduce the user data corresponding to this code word.
(c) If the error polynomial affecting the code word polynomial $c(x)$ in part (b) above, denoted $e(x)$, is given by:-

$$
e(x)=x^{6}+x^{2},
$$

show how the syndrome decoding method can correct these errors.
[11 marks]
Q.7. (a) Consider that a known signal $s(t)$ plus additive white Gaussian noise channel (AWGN) with power spectral density $\eta / 2 \mathrm{~W} / \mathrm{Hz}$ is the input to a linear timeinvariant filter followed by a sampler which samples the filter output at $t=T$. Show, using the usual notation, that the signal to noise ratio at the output of the sampler is governed by: -

$$
\left(\frac{S}{N}\right)_{o}=\frac{a^{2}(T)}{E\left[n_{O}^{2}(T)\right]} \leq \frac{2 E}{\eta}
$$

where $E$ denotes the energy content of $s(t)$. Note that the Schwarz inequality states that: -

$$
\left|\int_{-\infty}^{\infty} f_{1}(\omega) f_{2}(\omega) d \omega\right|^{2} \leq \int_{-\infty}^{\infty}\left|f_{1}(\omega)\right|^{2} d \omega \int_{-\infty}^{\infty}\left|f_{2}(\omega)\right|^{2} d \omega
$$

and, using the usual notation, the Fourier transform pair $f(t)$ and $f(\omega)$ are related by: -

$$
f(\omega)=\mathcal{F}[f(t)]=\int_{-\infty}^{\infty} f(t) e^{-j \omega t} d t
$$

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$$
f(t)=\mathcal{F}^{-1}[f(\omega)]=\frac{1}{2 \pi} \int_{-\infty}^{\infty} f(\omega) e^{j \omega t} d \omega .
$$

[10 marks]
(b) Summarise the principle characteristics of direct sequence spread spectrum (DSSS) communications.

