OLLSCOIL NA hÉIREANN, CORCAIGH
THE NATIONAL UNIVERSITY OF IRELAND, CORK

COLÁISTE NA hOLLSCOILE, CORCAIGH
UNIVERSITY COLLEGE, CORK

SUMMER EXAMINATIONS, 2005

B.E. DEGREE (ELECTRICAL)
B.E. DEGREE (MICROELECTRONIC)
M. ENG. SC. DEGREE (MICROELECTRONIC)

TELECOMMUNICATIONS
EE4004

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Time allowed: *3 hours*

Answer *five* questions.

The use of a Casio fx570w or fx570ms calculator is permitted.

Q.1 (a) A parabolic antenna with a noise temperature of 200 K and a gain of 38 dB is matched to a preamplifier with a noise temperature of 450 K and a power gain of 27 dB. The preamplifier is, in turn, matched to a receiver with a noise figure of 4 dB and an RF bandwidth of 50 MHz. If a similar antenna is used at a distance of 25 km to transmit a 6 GHz carrier at a power of 0.5 watts, calculate the carrier to noise ratio (in dB) at the receiver output.

[12 marks]

(b) Describe how non-linear quantization is used to minimize quantization noise and, in that context, discuss the A-law and μ-law implementations.

[8 marks]

Q.2 (a) Distinguish between *baud rate* and *bit rate* and, hence, discuss the factors which determine the information capacity of multi-level digital modulation schemes.

[*10 marks*]

(b) (i) Briefly describe the different switching procedures that are commonly used to route data through a Wide Area Network (WAN).

[4 *marks*]

(ii) Illustrate the format of an X.25 packet and briefly outline the function of each field in the packet.

[*6 marks*]

Q.3 (a) Illustrate the data and acknowledgement flow between two computers which are communicating over a dedicated link and which use the "stop and wait" ARQ scheme. From this derive an expression for the utilization of the link in the case where the link is prone to bit errors. It can be assumed that the only two significant time delays on the link are the propagation time and the packet (message) holding time.

[*10 marks*]

(b) For a given bit error probability, p, a given line date-rate, R, and a given propagation delay, τ_p, use the expression derived in part (a) of this question to determine the optimum packet size, n, to achieve the maximum line utilization.

[*8 marks*]

(c) For a data link as described in part (a) of this question, the line data rate is 10kbps, the propagation delay is 10ms and the bit error probability is 0.001. Determine the packet size needed to give maximum utilization.

[*2 marks*]

Q.4 Given that the $M \times M$ channel matrix $\left[P(Y|X)\right]$ for the *M*-ary uniform channel (MUC) with M input symbols, denoted $x_i,\ 1 \le i \le M$ and M output symbols, denoted $y_j,\ 1 \le j \le M$, is given by: -

$$\left[P(Y|X)\right] = \begin{bmatrix} 1-p & \alpha & \alpha & .. & \alpha \\ \alpha & 1-p & \alpha & .. & \alpha \\ \alpha & \alpha & 1-p & .. & \alpha \\ .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. \\ \alpha & \alpha & .. & \alpha & 1-p \end{bmatrix}$$

where $\alpha = \dfrac{p}{M-1}$ (i.e. all terms on the main diagonal equal $1-p$, all others equal α), show that: -

(a) The probability of receiving symbol $y_j,\ 1 \le j \le M$, denoted $P(y_j)$, is given by: -

$$P(y_j) = P(x_j)(1 - M\alpha) + \alpha$$

where $P(x_i)$ denotes the probability of sending symbol $x_i,\ 1 \le i \le M$.

[6 *marks*]

(b) $H[Y|X] = -((1-p)Log_2[1-p] + pLog_2[\alpha])$.

[8 *marks*]

(c) If the input symbols $x_i, 1 \le i \le M$ are equiprobable, show that the channel capacity of the MUC is given by: -

$$C_s = Log_2\left[(1-p)^{1-p}\alpha^p M\right] \; b/symbol.$$

[6 *marks*]

Q.5 A baseband digital communications system uses rectangular wave signaling with A_1 volts representing logic 1 and A_2 volts representing logic 0 (where $A_2 < A_1$). The receiver takes a single sample of the received signal during the bit signaling time and compares this sample with a decision threshold T. If the communications are affected by zero-mean additive Gaussian noise whose probability density function f_n is given by: -

$$f_n(v) = \frac{e^{-\frac{v^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}}$$

and P_0 and P_1 respectively denote the probability of sending logic 0 and logic 1, show that: -

(a) To minimize the resulting overall probability of error P_e, the threshold T is given by: -

$$T = \frac{A_1 + A_2}{2} + \frac{\sigma^2}{A_1 - A_2}\ln\left[\frac{P_0}{P_1}\right].$$

[10 *marks*]

(b) If $P_0 > P_1$ in (a) above, show that P_e is given by: -

$$P_e = \frac{1}{2}\left(1 - \left(P_0 erf\left[\frac{T - A_2}{\sqrt{2\sigma^2}}\right] + (1 - P_0)erf\left[\frac{A_1 - T}{\sqrt{2\sigma^2}}\right]\right)\right)$$

where: -

$$erf[x] = \frac{2}{\sqrt{\pi}}\int_0^x e^{-y^2}dy.$$

[10 *marks*]

Q.6 Given the following table of field elements of $GF(2^4)$: -

$$
\begin{aligned}
&0\\
&1\\
&\alpha\\
&\alpha^2\\
&\alpha^3\\
&\alpha^4 = \alpha \quad +1\\
&\alpha^5 = \alpha^2 + \alpha\\
&\alpha^6 = \alpha^3 + \alpha^2\\
&\alpha^7 = \alpha^3 + \alpha \quad +1\\
&\alpha^8 = \alpha^2 + 1\\
&\alpha^9 = \alpha^3 + \alpha\\
&\alpha^{10} = \alpha^2 + \alpha \quad +1\\
&\alpha^{11} = \alpha^3 + \alpha^2 + \alpha\\
&\alpha^{12} = \alpha^3 + \alpha^2 + \alpha \quad +1\\
&\alpha^{13} = \alpha^3 + \alpha^2 + 1\\
&\alpha^{14} = \alpha^3 + 1
\end{aligned}
$$

(a) Show that the generator polynomial for the $(15,7)$ double error correcting primitive BCH code based upon this field, denoted $g(x)$, is given by: -

$$g(x) = x^8 + x^7 + x^6 + x^4 + 1 .$$

[7 *marks*]

(b) If an error free non-systematic code word, denoted $c(x)$, is given by: -

$$c(x) = x^{11} + x^8 + x^7 + x^6 + x^3 + x^2,$$

deduce the user data corresponding to this code word.

[2 *marks*]

(c) If the error polynomial affecting the code word polynomial $c(x)$ in part (b) above, denoted $e(x)$, is given by: -

$$e(x) = x^{10} + x^2 ,$$

show how the syndrome decoding method can correct these errors.

[11 *marks*]

Q.7 (a) An analogue signal having an 10-kHz bandwidth is sampled at 1.5 times the Nyquist rate and each sample is quantised into one of 128 equally likely levels. Assuming that successive samples are statistically independent, the signal power at the receiver is 0.4 mW and the communication is affected by additive white Gaussian noise with power spectral density $\eta/2 = 10^{-11}\, W/Hz$, estimate via the use of a suitable graph, or otherwise, the minimum channel bandwidth required for error-free transmission of the information produced by this source.

[10 *marks*]

(b) Two DSSS systems, denoted A and B, possess the same probability of error (i.e. $P_e = Q\left[\sqrt{\frac{E_d}{2\eta}}\right]$) in an additive white Gaussian noise channel. The original information sequences are respectively represented by: -

System A **System B**

$$s_i(t) = \begin{cases} s_1(t) = A_1 & 0 \le t \le T \\ s_2(t) = 0 & 0 \le t \le T \end{cases} \qquad s_i(t) = \begin{cases} s_1(t) = A_2 & 0 \le t \le T \\ s_2(t) = -A_2 & 0 \le t \le T. \end{cases}$$

Deduce the ratio of the average signal energy per bit required by system A to that of system B.

[4 *marks*]

(c) If, instead, systems A and B of part (b) above are to possess the same average signal energy per bit, denoted E_b, show, using the table of values of $Q[z]$, or otherwise, that if the probability of error in system A is equal to 10 times that of system B, then we require: -

$$E_b \approx 4.05\eta$$

where, as usual, communications are affected by additive white Gaussian noise with power spectral density $\eta/2\ \ W/Hz$.

[6 *marks*]

Table of Values of $Q(z)$

z	$Q(z)$	z	$Q(z)$	z	$Q(z)$
0	0.5	1.7	0.0445655	3.4	0.000336929
0.05	0.480061	1.75	0.0400592	3.45	0.000280293
0.1	0.460172	1.8	0.0359303	3.5	0.000232629
0.15	0.440382	1.85	0.0321568	3.55	0.000192616
0.2	0.42074	1.9	0.0287166	3.6	0.000159109
0.25	0.401294	1.95	0.0255881	3.65	0.00013112
0.3	0.382089	2.	0.0227501	3.7	0.0001078
0.35	0.363169	2.05	0.0201822	3.75	0.0000884173
0.4	0.344578	2.1	0.0178644	3.8	0.000072348
0.45	0.326355	2.15	0.0157776	3.85	0.0000590589
0.5	0.308538	2.2	0.0139034	3.9	0.0000480963
0.55	0.29116	2.25	0.0122245	3.95	0.0000390756
0.6	0.274253	2.3	0.0107241	4.	0.0000316712
0.65	0.257846	2.35	0.00938671	4.25	10^{-5}
0.7	0.241964	2.4	0.00819754	4.75	10^{-6}
0.75	0.226627	2.45	0.00714281	5.2	10^{-7}
0.8	0.211855	2.5	0.00620967		
0.85	0.197663	2.55	0.00538615		
0.9	0.18406	2.6	0.00466119		
0.95	0.171056	2.65	0.00402459		
1.	0.158655	2.7	0.00346697		
1.05	0.146859	2.75	0.00297976		
1.1	0.135666	2.8	0.00255513		
1.15	0.125072	2.85	0.00218596		
1.2	0.11507	2.9	0.00186581		
1.25	0.10565	2.95	0.00158887		
1.3	0.0968005	3.	0.0013499		
1.35	0.088508	3.05	0.00114421		
1.4	0.0807567	3.1	0.000967603		
1.45	0.0735293	3.15	0.000816352		
1.5	0.0668072	3.2	0.000687138		
1.55	0.0605708	3.25	0.000577025		
1.6	0.0547993	3.3	0.000483424		
1.65	0.0494715	3.35	0.000404058		