# OLLSCOIL NA hÉIREANN, CORCAIGH <br> THE NATIONAL UNIVERSITY OF IRELAND, CORK <br> COLÁISTE NA hOLLSCOILE, CORCAIGH <br> UNIVERSITY COLLEGE, CORK 

SUMMER EXAMINATIONS, 2004

# B.E. DEGREE (ELECTRICAL) <br> B.E. DEGREE (MICROELECTRONIC) <br> M. ENG. SC. DEGREE (MICROELECTRONIC) 

# TELECOMMUNICATIONS <br> EE4004 

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Time allowed: 3 hours

Answer six questions.
The use of a Casio fx 570 w or fx570 ms calculator is permitted.
Q.1. (a) What factors influence the noise performance of a satellite receiving system?
[7 marks]
A satellite in geostationary orbit is equipped with a 40 W transmitter and employs a 1 m diameter parabolic antenna to transmit a 10.5 GHz signal to earth. Estimate the approximate signal power density within the satellite footprint and, hence, the approximate signal power level in dBm available from a 1.8 m diameter ground station antenna.
[6 marks]
(b) In the context of PCM systems, what do you understand by $\mu$-law and A-law companding?
[7 marks]
Q.2. (a) Describe the network architecture of the GSM mobile telephone system.
(b) (i) Illustrate the format of an ATM cell and briefly describe the function of each field in the cell.
(ii) Describe how the cell boundaries are identified in an ATM system.
[3 marks]
(iii) Draw a state diagram to illustrate how synchronization is achieved at an ATM network node.
Q.3. (a) Illustrate the operation of a store and forward packet data network considering source and destination nodes and two intermediate nodes (nodes 1 and 2). From this, determine formulas for the network delay (ND) and total message transmission time (TT). Ignore propagation delay.
[12 marks]
(b) A message of 1000 bits is to be sent over a data network from source to destination via two intermediate nodes. Each link along the route has a datarate of 1 Mbps and is totally dedicated to this transmission. When the message is broken into packets, a header size of 100 bits is used in all cases. Calculate the network delay and the total transmission time if the message is subdivided into the following numbers of packets: -
(i) 1
(ii) 5
(iii) 10

Comment on the trend indicated by these results.
Q.4. Given that the channel matrix $\left[P\left(Y_{1} \mid X\right)\right]$ for the binary symmetric channel (BSC) with probability of error $p(p \neq 1 / 2)$ satisfies: -

$$
\begin{aligned}
{\left[P\left(Y_{1} \mid X\right)\right] } & =\left[\begin{array}{cc}
1-p & p \\
p & 1-p
\end{array}\right] \\
& =\underbrace{\left[\begin{array}{cc}
1 & -1 \\
1 & 1
\end{array}\right]}_{F} \underbrace{\left[\begin{array}{cc}
1 & 0 \\
0 & 1-2 p
\end{array}\right]}_{D} \underbrace{\left[\begin{array}{cc}
1 / 2 & 1 / 2 \\
-1 / 2 & 1 / 2
\end{array}\right]}_{F^{-1}}
\end{aligned}
$$

where $D$ is a diagonal matrix and the columns of $F$ are eigenvectors of $\left[P\left(Y_{1} \mid X\right)\right]$, show that if $n$ such BSCs are connected in series (i.e. the outputs of BSC $i$ become the inputs of BSC $i+1,1 \leq i \leq n-1$ ), then: -
(a) The composite channel matrix $\left[P\left(Y_{n} \mid X\right)\right]$ is given by: -

$$
\left[P\left(Y_{n} \mid X\right)\right]=\left[\begin{array}{cc}
1-p_{c} & p_{c} \\
p_{c} & 1-p_{c}
\end{array}\right]
$$

where $p_{c}$, the probability of error in the composite channel, is given by:

$$
p_{c}=\frac{1-(1-2 p)^{n}}{2} .
$$

(b) Given that the maximum entropy of an $m$ symbol source is given by $\log _{2}[m]$, the composite channel capacity $C_{s}^{c}$ is given by: -

$$
C_{s}^{c}=1+\frac{1}{2}\left(\left(1-(1-2 p)^{n}\right) \log _{2}\left[\frac{1-(1-2 p)^{n}}{2}\right]+\left(1+(1-2 p)^{n}\right) \log _{2}\left[\frac{1+(1-2 p)^{n}}{2}\right]\right) .
$$

(c) If $p$ is sufficiently small such that its square and higher powers can be neglected, show that the composite channel capacity $C_{s}^{c}$ is approximately that of a single BSC with probability of error $n \times p$. Note that: -

$$
(x+y)^{n}=\sum_{i=0}^{n}\binom{n}{i} x^{i} y^{n-i} \quad \text { and } \quad\binom{n}{i}=\frac{n!}{i!(n-i)!} .
$$

[4 marks]
Q.5. Consider a binary digital modulation scheme with probability of error $p_{e}$ that sends data frames containing $k$ message bits (with no error correction). If the system is subsequently enhanced via the addition of an $(n, k)$ linear block code capable of correcting up to $t$ errors in any $n$ bit frame, then: -
(a) Show that, having implemented the linear block code, the probability of failing to correctly recover the transmitted frame at the receiver is reduced (relative to the initial system) by a factor $\varepsilon$ where: -

$$
\varepsilon=\frac{1-\left(1-p_{e}\right)^{k}}{1-\sum_{i=0}^{t}\binom{n}{i} p_{e}^{i}\left(1-p_{e}\right)^{n-i}},
$$

i.e. the $(n, k)$ linear block code is $\varepsilon$ times less likely to fail than the original system.
[8 marks]
(b) If $k=7$ and $t=1$, using a graph (or otherwise), estimate the value of $p_{e}$ resulting in $\varepsilon=50$.
[6 marks]
(c) Specify a suitable parity check matrix $\underline{H}$ if $k=7$ and $t=1$ and show that syndrome decoding fails if the first and third bits of any codeword are corrupted by noise.
Q.6. A baseband digital communications system uses rectangular wave signaling with $A_{1}$ volts representing logic 1 and $A_{2}$ volts representing logic 0 (where $A_{2}<A_{1}$ ). The receiver takes a single sample of the received signal during the bit signaling time and compares this sample with a decision threshold $T$. If the communications are affected by zero-mean additive Gaussian noise whose probability density function $f_{n}$ is given by: -

$$
f_{n}(v)=\frac{e^{-\frac{v^{2}}{2 \sigma^{2}}}}{\sqrt{2 \pi \sigma^{2}}}
$$

and $P_{0}$ and $P_{1}$ respectively denote the probability of sending logic 0 and logic 1 , show that: -
(a) To minimize the resulting overall probability of error $P_{e}$, the threshold $T$ is given by: -

$$
T=\frac{A_{1}+A_{2}}{2}+\frac{\sigma^{2}}{A_{1}-A_{2}} \ln \left[\frac{P_{0}}{P_{1}}\right] .
$$

[10 marks]
(b) If we require $T=0$ volts and $P_{0}=P_{1}$ in (a) above, show that $P_{e}$ is given by: -

$$
P_{e}=\frac{1}{2}\left(1-e r f\left[\frac{A_{1}}{\sigma \sqrt{2}}\right]\right)
$$

where: -

$$
\operatorname{erf}[x]=\frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-y^{2}} d y
$$

and noting that $\operatorname{erf}[\infty]=1$.
[10 marks]
Q.7. (a) Consider that a known signal $s(t)$ plus additive white Gaussian noise channel (AWGN) with power spectral density $\eta / 2 \mathrm{~W} / \mathrm{Hz}$ is the input to a linear timeinvariant filter followed by a sampler which samples the filter output at $t=T$. Show, using the usual notation, that the signal to noise ratio at the output of the sampler is governed by: -

$$
\left(\frac{S}{N}\right)_{o}=\frac{a^{2}(T)}{E\left[n_{O}^{2}(T)\right]} \leq \frac{2 E}{\eta}
$$

where $E$ denotes the energy content of $s(t)$.
(b) Summarise the principle characteristics of direct sequence spread spectrum (DSSS) communications.
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