## AUTUMN EXAMINATIONS, 2003

## B.E. DEGREE (ELECTRICAL)

## TELECOMMUNICATIONS

EE4004
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Time allowed: 3 hours
Answer six questions.
The use of a Casio fx570w or fx570ms calculator is permitted.

1. (a) Discuss the relationship between quantisation noise and the number of quantisation levels in pulse code modulation (PCM).
[6 marks]
(b) A microwave link using 64 QAM modulation is used to carry three signals of 5 $\mathrm{MHz}, 10 \mathrm{MHz}$ and 15 MHz bandwidth respectively, which have been encoded using PCM. If the carrier modulation rate is $6 \times 10^{7}$ phase changes per second, what is the maximum possible signal to quantisation noise ratio in dB ?
[10 marks]
2. (a) Compare the capacities of BPSK, QPSK, and QAM systems.
[8 marks]
(b) (i) Illustrate the timing of the packet and acknowledgement transfers for a data-link which uses an "alternating-bit" ARQ scheme and from this determine an expression for the utilization, U , of the data-link when the data-link is error-free and also when the data-link has errors.
[6 marks]
(ii) A data link is 100 km long and uses a data rate of 150 Mbps . It uses a data packet size of 2000 bits, and an acknowledgement size of 50 bits. The propagation delay of the line is $5 \mu \mathrm{~s} / \mathrm{km}$. Determine the utilization of the data-link when the link is error free and also when the link has a bit error probability of 0.01 .
3. (a) Illustrate and briefly describe the three common network topologies which are used for local area networks (LANs).
(b) For LANs using Ethernet protocols describe the following:
(i) The CSMA/CD algorithm.
(ii) The truncated binary exponential back-off algorithm.
(c) An Ethernet LAN has a data rate of 10 Mbps and uses a frame size of 1000 bits. Determine the maximum length of the LAN if the speed of propagation is $2 \times 10^{8} \mathrm{~m} / \mathrm{s}$.
4. The channel diagram below represents the "binary erasure channel": -


Binary Erasure Channel Diagram
The output $y_{2}=e$ indicates an "erasure" - that is, the output is in doubt and should be: erased. Show that if we consider $Y$ to be a source generating symbols $y_{0}, y_{1}$ and $e$ : with probabilities appropriate to the channel diagram then: -
(a) $\quad H(Y)=(1-p)\left((\alpha-1) \log _{2}[1-\alpha]-\alpha \log _{2}[\alpha]-\log _{2}[1-p]\right)-p \log _{2}[p]$.
[6 marks]
(b) $\quad H(Y \mid X)=-p \log _{2}[p]-(1-p) \log _{2}[1-p]$.
[6 marks]
The channel capacity $C_{s}=1-p$.
[5 marks]
5. (a) For a linear block code (and assuming the usual notation), prove that the syndrome $s$ is the sum (modulo 2) of those rows of $H^{T}$ (where $A^{T}$ denotes the transpose of the matrix $A$ ) corresponding to the error locations in the received vector $r$.
[6 marks]
Consider a single-error-correcting linear block code with the following $H$ matrix: -

$$
H=\left[\begin{array}{lllllllllll}
1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1
\end{array}\right] .
$$

(i) Deduce the code vector $c$ corresponding to the data vector: -

$$
d=\left[\begin{array}{llllll}
1 & 1 & 0 & 0 & 0 & 1
\end{array}\right]
$$

[3 marks]
(ii) Show how the error correction process can remove the effect of the following error pattern $e_{1}$ on the code vector $c$ of part (i) above: -

$$
e_{1}=\left[\begin{array}{llllllllll}
0 & 0 & & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right] .
$$

[4 marks]
(iii) Illustrate the failure of the linear block code to remove the effects of the error pattern $e_{2}$ on any code vector $c$ of this linear block code: -

$$
e_{2}=\left[\begin{array}{lllllllllll}
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{array}\right]
$$

6. Typical expressions for ASK and PSK modulated waveforms representing binary data, where in each case $T$ is an integer times $1 / f_{c}$, are as follows: -

$$
\begin{aligned}
& \text { ASK PSK } \\
& s_{i}(t)=\left\{\begin{array}{ll}
s_{1}(t)=A_{1} \cos \left[\omega_{c} t\right] & 0 \leq t \leq T \\
s_{2}(t)=0 & 0 \leq t \leq T
\end{array} \quad s_{i}(t)= \begin{cases}s_{1}(t)=A_{2} \cos \left[\omega_{c} t\right] & 0 \leq t \leq T \\
s_{2}(t)=-A_{2} \cos \left[\omega_{c} t\right] & 0 \leq t \leq T\end{cases} \right.
\end{aligned}
$$

In addition, the probability of error for a binary modulation scheme (denoted MOD) with optimum detection in the presence of AWGN with a power spectral density of $\eta / 2 \mathrm{~W} / \mathrm{Hz}$ is given by: -

$$
\left.P_{e}^{M O D}=Q \sqrt{\frac{E_{d}}{2 \eta}}\right]
$$

where $E_{d}$ denotes the energy difference in the appropriate signal (over a single bit interval).
(a) Derive expressions for $P_{e}^{A S K}$ and $P_{e}^{P S K}$
(b) If the average signal energy per bit for the ASK and PSK modulation schemes above is made equal, derive the following expression for the enhancement in reliability, denoted $E$, achieved by choosing PSK over ASK when both schemes deliver the same bit rate: -

$$
E=\frac{P_{e}^{A S K}}{P_{e}^{P S K}}=\frac{\left\{\sqrt{\frac{A_{2}^{2} T}{2 \eta}}\right]}{\left.q \sqrt{\frac{A_{2}^{2} T}{\eta}}\right] .}
$$

(c) Using the table of values of $Q[z]$ provided to draw a suitable graph, or otherwise, estimate the amplitude $A_{2}$ resulting in $E=45$ when $\eta / 2=10^{-12} \mathrm{~W} / \mathrm{Hz}$ and the bit rate is $1 \mathrm{Mb} / \mathrm{s}$.
(a) An analogue signal having an $8-\mathrm{kHz}$ bandwidth is sampled at 1.25 times the Nyquist rate and each sample is quantised into one of 64 equally likely levels. Assuming that successive samples are statistically independent, the signal power at the receiver is 0.2 mW and the communication is affected by additive white Gaussian noise with power spectral density $\eta / 2=10^{-12} \mathrm{~W} / \mathrm{Hz}$, estimate via the use of a suitable graph, or otherwise, the minimum channel bandwidth required for error-free transmission of the information produced by this source.
[7 marks]
(b) Summarise the principle characteristics of spread spectrum communications.
[10 marks]

## Table of Values of $Q(z)$

| $z$ | $Q(z)$ | $z$ | $Q(z)$ |
| :---: | :---: | :---: | :---: |
| 0 | 0.5 | 1.7 | 0.0445655 |
| 0.05 | 0.480061 | 1.75 | 0.0400592 |
| 0.1 | 0.460172 | 1.8 | 0.0359303 |
| 0.15 | 0.440382 | 1.85 | 0.0321568 |
| 0.2 | 0.42074 | 1.9 | 0.0287166 |
| 0.25 | 0.401294 | 1.95 | 0.0255881 |
| 0.3 | 0.382089 | 2. | 0.0227501 |
| 0.35 | 0.363169 | 2.05 | 0.0201822 |
| 0.4 | 0.344578 | 2.1 | 0.0178644 |
| 0.45 | 0.326355 | 2.15 | 0.0157776 |
| 0.5 | 0.308538 | 2.2 | 0.0139034 |
| 0.55 | 0.29116 | 2.25 | 0.0122245 |
| 0.6 | 0.274253 | 2.3 | 0.0107241 |
| 0.65 | 0.257846 | 2.35 | 0.00938671 |
| 0.7 | 0.241964 | 2.4 | 0.00819754 |
| 0.75 | 0.226627 | 2.45 | 0.00714281 |
| 0.8 | 0.211855 | 2.5 | 0.00620967 |
| 0.85 | 0.197663 | 2.55 | 0.00538615 |
| 0.9 | 0.18406 | 2.6 | 0.00466119 |
| 0.95 | 0.171056 | 2.65 | 0.00402459 |
| 1. | 0.158655 | 2.7 | 0.00346697 |


| $z$ | $Q(z)$ |
| :--- | :--- |
| 3.4 | 0.000336929 |
| 3.45 | 0.000280293 |
| 3.5 | 0.000232629 |
| 3.55 | 0.000192616 |
| 3.6 | 0.000159109 |
| 3.65 | 0.00013112 |
| 3.7 | 0.0001078 |
| 3.75 | 0.0000884173 |
| 3.8 | 0.000072348 |
| 3.85 | 0.0000590589 |
| 3.9 | 0.0000480963 |
| 3.95 | 0.0000390756 |
| 4. | 0.0000316712 |
| 4.25 | $10^{-5}$ |
| 4.75 | $10^{-6}$ |
| 5.2 | $10^{-7}$ |
|  |  |


| 1.05 | 0.146859 |
| :--- | :--- |
| 1.1 | 0.135666 |
| 1.15 | 0.125072 |
| 1.2 | 0.11507 |
| 1.25 | 0.10565 |
| 1.3 | 0.0968005 |
| 1.35 | 0.088508 |
| 1.4 | 0.0807567 |
| 1.45 | 0.0735293 |
| 1.5 | 0.0668072 |
| 1.55 | 0.0605708 |
| 1.6 | 0.0547993 |
| 1.65 | 0.0494715 |$\quad$| 2.75 | 0.00297976 |
| :--- | :--- |
| 2.8 | 0.00255513 |
| 2.9 | 0.00218596 |
| 2.95 | 0.00186581 |
| 3. | 0.00158887 |
| 3.05 | 0.0013499 |
| 3.1 | 0.00114421 |
| 3.15 | 0.000967603 |
| 3.2 | 0.000687138 |
| 3.25 | 0.000577025 |
| 3.3 | 0.000483424 |

